# VECTOR INTEGRATION

* $\vec{F}(\vec{r})$ = a continuous vector point function.

* $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

* Let $\vec{r} = f(t)$ be a continuously differentiab[le] curve C.

$\therefore \dfrac{d\vec{r}}{ds}$ = a unit vector function along the tangent at any point P on the curv[e]

$$\therefore \int_C \vec{F} \cdot \frac{d\vec{r}}{ds}\, ds = \int_C \vec{F} \cdot d\vec{r}$$

* ~~Let~~

1. Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where

$\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$ and curve C is $\vec{r} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$, t varies from $-1$ to $1$.

Soln: The eqn of curve in parametric form is

$$x = t,\ y = t^2,\ z = t^3$$

$[\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ given.

$\therefore x = t,\ y = t^2,\ z = t^3]$

Now, $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$

$= t.t^2\vec{i} + t^2.t^3\vec{j} + t^3.t\vec{k}$

$= t^3\vec{i} + t^5\vec{j} + t^4\vec{k}$

Also $\dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(t\vec{i} + t^2\vec{j} + t^3\vec{k}) = \vec{i} + 2t\vec{j} + 3t^2\vec{k}$

$\therefore \vec{F}\cdot\dfrac{d\vec{r}}{dt} = (t^3\vec{i} + t^5\vec{j} + t^4\vec{k})\cdot(\vec{i} + 2t\vec{j} + 3t^2\vec{k})$

$= t^3 + 2t^6 + 3t^6 = t^3 + 5t^6$ —(1)

$\therefore \int_C \vec{F}\cdot d\vec{r} = \int_C \vec{F}\cdot\dfrac{d\vec{r}}{dt}\,dt = \int_{-1}^{1}(t^3 + 5t^6)\,dt$ [using (1)]

$= \left[\dfrac{t^4}{4} + \dfrac{5t^7}{7}\right]_{-1}^{1} = \left(\dfrac{1}{4} + \dfrac{5}{7}\right) - \left(\dfrac{1}{4} - \dfrac{5}{7}\right) = \dfrac{10}{7}$